

# Playing the Aharon-Vaidman quantum game with a Young type photonic qutrit

Piotr Kolenderski,<sup>1,2,\*</sup> Urbasi Sinha,<sup>1</sup> Li Youning,<sup>3</sup> Tong Zhao,<sup>1</sup> Mathew Volpini,<sup>1</sup> Adán Cabello,<sup>4,5</sup> Raymond Laflamme,<sup>1</sup> and Thomas Jennewein<sup>1</sup>

<sup>1</sup>*Institute for Quantum Computing, University of Waterloo,  
200 University Ave. West, Waterloo, Ontario, CA N2L 3G1*

<sup>2</sup>*Institute of Physics, Nicolaus Copernicus University, Grudziadzka 5, 87-100 Toruń, Poland*

<sup>3</sup>*Department of Physics, Tsinghua University, Beijing 100084, P. R. China*

<sup>4</sup>*Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain*

<sup>5</sup>*Department of Physics, Stockholm University, S-10691 Stockholm, Sweden*

(ΩDated: August 1, 2011)

The Aharon-Vaidman game exemplifies the advantage of using simple quantum systems to outperform classical strategies. We present an experimental test of this quantum advantage by using a three-state quantum system (qutrit) encoded in a spatial mode of a single photon passing through a system of three slits, which we call Young type qutrit. We prepared its states by controlling the photon propagation and the number of slits that are open, and performed POVM-measurements by placing detectors in the positions corresponding to near and far field. These tools allowed us to perform tomographic reconstructions of qutrit states and play the game with compelling evidence of the quantum advantage.

PACS numbers:

## INTRODUCTION

One of the basic requirements of effective quantum computation [1, 2], quantum communication [3, 4], quantum metrology [5] and quantum games [6] is the ability manipulate multidimensional quantum states. The experimental realization of such scenarios is a demanding task. Much effort has been channelled towards extension of accessible multidimensional quantum states. The recent highest multidimensional states are based on e.g. two superconducting qubits [7], six photons [8] or twelve molecular nuclear spins [9]. These experimental approaches are challenging as they resort to weak nonlinear interactions in case of entangled photons or very low temperatures in case of superconductivity and nuclear spins. Thus, it is of primary importance to establish a simple method to encode high-dimensional quantum states. One such approach, where the 7 and 8 dimensional state has been encoded in a single photon degree of freedom has been shown by Lima et al in Refs. [10, 11]. Another approach to generate, control and measure qutrits pursued in Ref. [12] exploited spontaneous parametric down converted photons and unbalanced 3-arm fiber optic interferometers in a scheme analogous to the Franson interferometric arrangement for qubits. A Bell-type test for energy-time entangled qutrits was possible within this scenario [13].

We present here an even simpler way of preparing and measuring pure qutrit states and encoding them in a single photon spatial degree of freedom within the scenario similar to the Young slits experiment [14, 15]. The setup comprising of single photon source, triple slit [16] and single photon

detector allows us to perform the qutrit quantum tomography as well as demonstrate the quantum game proposed by Aharon and Vaidman [6].

## CONCEPT

We will begin with the introduction of the concept of the qutrit encoded in spatial degrees of freedom of a single photon. In the next step we discuss the Aharon-Videman quantum game and its experimental implementation.

### The qutrit

Let us introduce the concept of a qutrit encoded into a single photon spatial degree of freedom. This is done using triple slits and a single photon source as depicted in Fig. 1. The photon's initial spatial mode is prepared in gaussian with the characteristic diameter much larger than the size of the slits and with the peak intensity coincident with the slits area. Under these conditions we can consider the state of a photon to be a plane wave. Hence, passing through the slits the spatial wave function can be written as  $|\psi\rangle = \int_{-\infty}^{\infty} dx S(x) \exp(i\mathbf{k}\mathbf{r}) |x\rangle$ , where  $\mathbf{k} = (k_x, k_z)$  is the wave vector of length  $k = 2\pi/\lambda$ ,  $\mathbf{r} = (x, z)$  and  $S(x)$  stands for the transmission probability amplitude, which is constant on each slit and  $S(x) = 0$  elsewhere. This means that the wave function comprises of three contributions originating from the three slits. Each of them can be written in momentum representation as:

$$|n\rangle = \int dk_x \sqrt{\frac{a}{2\pi}} \text{sinc}\left(\frac{k_x a}{2}\right) e^{-ink_x d} |k_x\rangle, \quad (1)$$

where  $n = 0, 1, 2$  denotes slit number and  $d$  is a distance between the slits. We assume here that  $a$  is very small, which allows us to approximate a factor as constant. This definitions allow us to write the state of the transmitted photon as:

$$|\psi\rangle = \frac{1}{\sqrt{3}} (a|0\rangle + b|1\rangle + c|2\rangle), \quad (2)$$

which accounts for the basic definition of a Young type qutrit. Here amplitudes  $a$ ,  $b$  and  $c$  depend on the transmission function  $S(x)$ .

Let us now move to the description of the projective measurements. These are determined by the laws of propagation and the geometry of the setup. For simplicity, we chose to detect in the positions corresponding to near and far field. This can be done using the lens and placing a detector in the focal plane (far field) and in the plane where the image of the slits is formed (near field).

Hence, for the near field, if the active area of the detector is larger than the image of each slit, the detection in the position of the slit number  $n$  corresponds to the following measurement:

$$M_{\text{nf}}(n) = \mu_{\text{nf}} |n\rangle \langle n|, \quad (3)$$

where  $n = 0, 1, 2$ ,  $\mu_{\text{nf}}$  is the normalization factor to be specified later and subscript nf stands for near field.

The interpretation of measurements in the far field needs more attention. A detection in the position  $x$  in the focal plain corresponds to the projector onto  $|k_x\rangle$ , which is related to the plane wave propagating in the direction given by the transverse wave vector  $k_x$ . The laws of geometrical optics allow us to find the relation between the position  $x$  in the far field plain and transverse wave vector  $k_x$ :  $x = f k_x / k$ , where  $f$  is focal length. Hence the probability to detect a photon  $|\langle \psi | k_x \rangle|^2$  can be seen as proportional to  $|\sqrt{\frac{a}{2\pi}} \text{sinc}(\frac{1}{2} k_x a) \langle \phi(k_x d) | \psi \rangle|^2$ , where we introduced  $|\phi(\theta)\rangle = |0\rangle + \exp(i\theta)|1\rangle + \exp(i2\theta)|2\rangle$ . Based on this observation we can define the measurement as:

$$M_{\text{ff}}(\theta) = \mu_{\text{ff}}(\theta) |\phi(\theta)\rangle \langle \phi(\theta)|, \quad (4)$$

where  $\mu_{\text{ff}}(\theta)$  is the normalization factor and subscript ff stands for far field. The phase parameter  $\theta$  has a direct relation with the position  $x$  of the detector in far field plain,  $\theta = 2\pi dx/f$ .

Now we move on to the description applications of the Young type qutrit. First the measurements operators will be utilized to construct a POVM and perform pure state tomography. Next we will use them again to simulate the Aharon-Vaidman quantum game.

### Tomographic reconstruction of the state

Two types of measurements introduced above can be used to construct POVM, which is topographically complete for pure states. For this reason we take three near field measurements  $M_{\text{nf}}(n)$ ,  $n = 0, 1, 2$  and six far field operators  $M_{\text{ff}}(\theta)$  corresponding to  $\theta = \{0, \pi, 2\pi/3, -2\pi/3, 5\pi/3, -5\pi/3\}$ . This specific choice requires renormalization which can be done putting when  $6\mu_{\text{ff}}(\theta) = \mu_{\text{nf}} = 1/2$ . It is easy to see that those measurements comprise rank 7 POVM set allowing for reconstruction of arbitrary pure state. This is the consequence of a well know fact in quantum mechanics that a pure state of a one dimensional system can be reconstructed based on its intensity and the intensity of its Fourier transform, see for instance [17]. Here the intensity of the state corresponds to near field and its Fourier transform to the far field measurement.

It is also noteworthy that using only far field operators, one can reconstruct the phases but in general it is not possible to distinguish between the states with equal amplitudes of the form eg.  $a|0\rangle + b|1\rangle$  and  $a|1\rangle + b|2\rangle$ . Hence it is necessary to take into account the measurements in the near field as those give the direct information about the magnitudes of amplitudes.

### Quantum game

The ability to encode and measure qutrit states can be utilized to demonstrate Aharon and Vaidman's quantum game [6]. This is a three turn game which can be summarized as follows: I) Alice begins the game by preparing a single particle that she places inside two boxes labelled 0 and 2 or any other place other than the two boxes, which we can be denoted by 1. II) Bob, who has no information about the state of the system, can look for the particle in box 0 or 2. His objective

is to leave no trace of his action, so he tries to leave the box exactly as it was before. He is not allowed to touch the box that he chooses not to open. III) Alice gets access to the boxes and can perform any measurement she wants. She then has the option of either cancelling or accepting this trial of the game. She wins if she accepts a trial in which Bob found the particle and loses if she accepts a trial in which Bob did not find it. Alice's objective is to maximize the probability of the trials she does not cancel in which Bob finds the particle. It is assumed that both parties play fair.

A classical strategy allows for at most 50% chance to win. On the other hand, when Alice uses quantum particles her chance rises above this limit and ideally reaches 100%. Let us see how this can be done assuming that Alice chooses her initial state to be  $|\psi_A\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle)$ . Now Bob performs a projective measurement on state number 0. If he finds the particle, then his state becomes  $|\psi_B^{(p)}\rangle = |0\rangle$ , otherwise  $|\psi_B^{(n)}\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$ . In the third turn of the game Alice makes a projective measurement on  $|\psi_{Am}\rangle = \frac{1}{\sqrt{3}}(|0\rangle - |1\rangle + |2\rangle)$ . If she detects a particle, she accepts the game trial, and if she does not, she cancels it. Now it is clear that Alice cannot lose as whenever Bob does not detect a photon, the state after the second turn is  $|\psi_B^{(n)}\rangle$  and Alice's measurement always gives the same result  $\langle\psi_B^{(n)}|\psi_{Am}\rangle = 0$ . The same holds if Bob chooses the box number 2.

## EXPERIMENT

The experimental setup is depicted in Fig. 1. We used two single photon sources: heralded parametric down conversion source based on collinear PPKTP crystal (PDC-SPS) and attenuated intensity stabilized HeNe laser (AL-SPS). The wavelengths of the generated photons were  $\lambda = 810$  nm and  $\lambda = 632$  nm respectively for PDC-SPS and AL-SPS. Coupling the photons into single mode fibers (SMF) and using a standard optical system allowed us to fulfil the assumption of the plane wave incidence at the slits by setting the characteristic mode diameter to approximately 3 mm. Rotating the mirror M and changing the number of opened slits using blocking mask B allowed us to control the state of the qutrit. Under this simplifying assumptions the experimentally possible states are in the following form  $|\psi\rangle = (|0\rangle + e^{ikd \sin \alpha} |1\rangle + e^{2ikd \sin \alpha} |2\rangle) / \sqrt{3}$ .

For the measurement part we placed a lens of 2 inch diameter, which allowed us to perform far and near field measurements in the transverse planes at distances of 150 mm and 326 mm, respectively. Large 2 inch pellicle beamsplitter BS was introduced to reduce the disturbance of the setup while changing the planes of a detection. Each detector system (D1,D2) comprised of multimode fiber mounted on precise motorized stage and Perkin Elmer avalanche photodiode. Newport CMA-25CCCL and Thorlabs ZST13 motors allowed us to control the transverse position of the fiber with an accuracy of approximately 1  $\mu$ m. Counts were registered by FPGA logic system.

To perform measurements either for the tomographic reconstruction or the quantum game we need to know the specific position of the detectors, which can be computed or measured. First we checked how theoretical predictions conform to experimental data. For that we prepared the state  $|0\rangle + |1\rangle + |2\rangle$  opening all slits and setting the initial direction of photon propagation to  $\alpha = 0$ . Next we measured the photon count rates as a function of the detector position in far and near field planes. The results together with the best fits are presented as insets in Fig. 1. Blue (online) dots

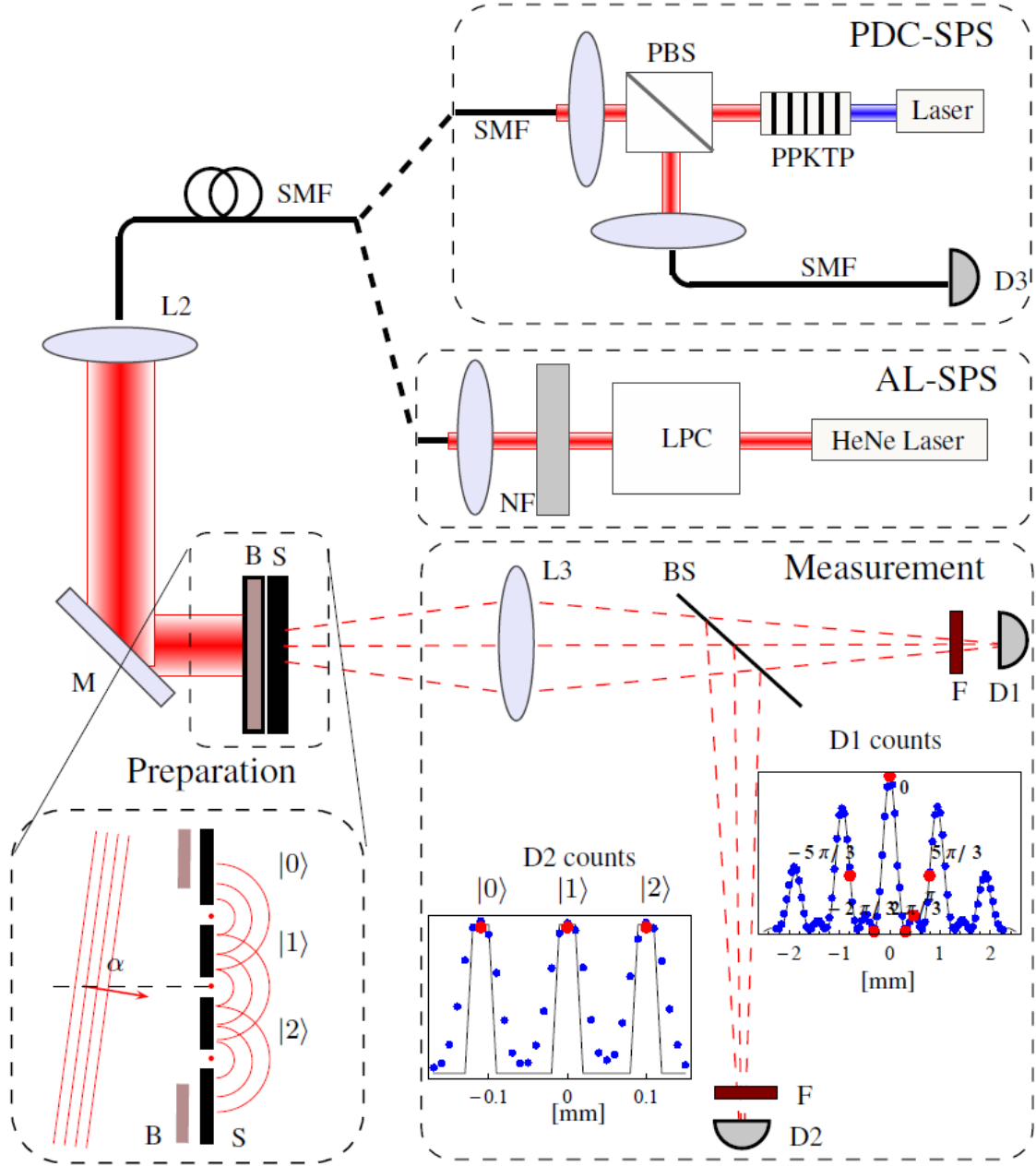


FIG. 1: Experimental setup. The attenuated laser single photon source (AL-SPS) comprise of HeNe laser, laser power controller (LPC) and neutral filter (NF). The heralded single photon source (PDC-SPS) is based on PPKTP crystal pumped by blue continues wave laser. Heralding photon is detected by detector D3. The single photons from both sources are coupled to single mode fibera (SMF). A qutrit is prepared using the blocking mask and three slits. Next the measurement part of the setup comprise of 2 inch diameter  $f = 150\text{mm}$  lens (L), 2 inch diameter pellicle beamsplitter (BS), color filters (F) and two detection systems (D1, D2), each comprised of multimode fiber mounted on precise motorized stage and Perkin Elemer avalanche photodiode.

Measurement setting \ Input state	$ \psi_1\rangle$	$ \psi_2\rangle$	$ \psi_3\rangle$
$M_{\text{ff}}(-5\pi/3)$	0.097(1)	0.107(1)	0.028(1)
$M_{\text{ff}}(-2\pi/3)$	0.0017(1)	0.056(1)	0.034(1)
$M_{\text{ff}}(0)$	0.259(1)	0.177(1)	0.176(1)
$M_{\text{ff}}(2\pi/3)$	0.0014(1)	0.040(1)	0.049(1)
$M_{\text{ff}}(\pi)$	0.031(1)	0.0010(1)	0.178 (1)
$M_{\text{ff}}(5\pi/3)$	0.108(2)	0.117(2)	0.040(1)
$M_{\text{nf}}(0)$	0.167(1)	0.0027(1)	0.0030(1)
$M_{\text{nf}}(1)$	0.167(1)	0.260(1)	0.258(1)
$M_{\text{nf}}(2)$	0.165(1)	0.237(1)	0.240(1)

TABLE I: Measured photon detection probabilities using attenuated laser source.

represent experimental data, continuous line is a theoretical fit. The position corresponding to the far field part of POVM are marked with bigger red dots on inset next to detector D1. Analogically the positions in near field are marked in inset next to detector D2. Note that the smoothed shape of the slits image (near field) is attributed to the fact that the detector dimension was large compared to the slits characteristic size.

### Characterization of prepared states

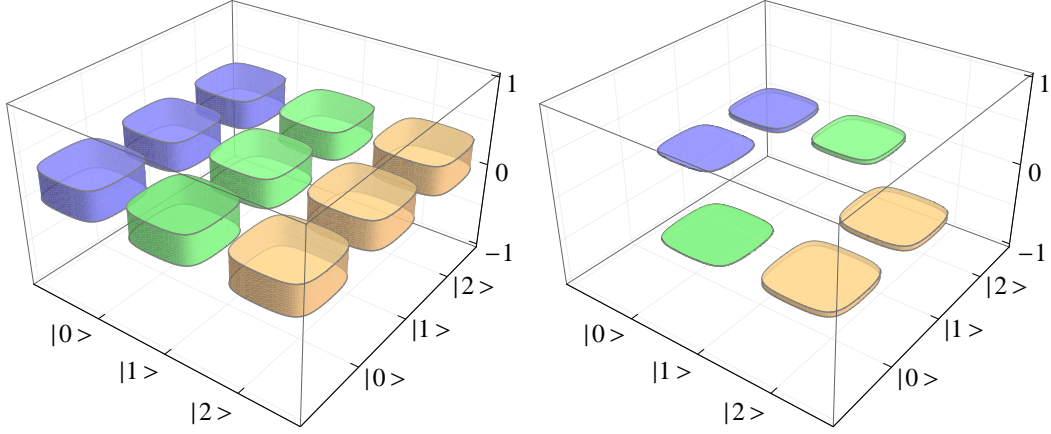
First we perform a tomographic reconstruction measuring the photon counts placing detectors in positions described above. For the demonstration and the justification of the results of the quantum game we take three typical states:  $|\psi_1\rangle = |0\rangle + |1\rangle + |2\rangle$ ,  $|\psi_2\rangle = |1\rangle + |2\rangle$ ,  $|\psi_3\rangle = |1\rangle + |2\rangle \exp(i\beta)$ . For the first state all slits were open, for the second slit number 0 was closed and for the last state the propagation direction of the photon emerging from the fiber was modified in order to introduce a phase  $\beta$ . This measurement was done using AL-SPS, its outcomes are gathered in table I and the results of tomographic reconstruction using maximal likelihood method [18] are depicted in Fig. 2.

It is seen in Fig. 2(a,b) that for the states  $|\psi_1\rangle$  and  $|\psi_2\rangle$  the real part dominates as there is no phase present. On the other hand changing the initial plane wave direction  $\alpha$  it is possible to introduce the phase what is apparent in Fig. 2(c) as the imaginary bars are significant. Ideally, the imaginary part of density matrix for states  $|\psi_1\rangle$  and  $|\psi_2\rangle$  is zero. Here the nonzero height is attributed to the noise originating from dark counts and stray light as well as non-perfect positioning.

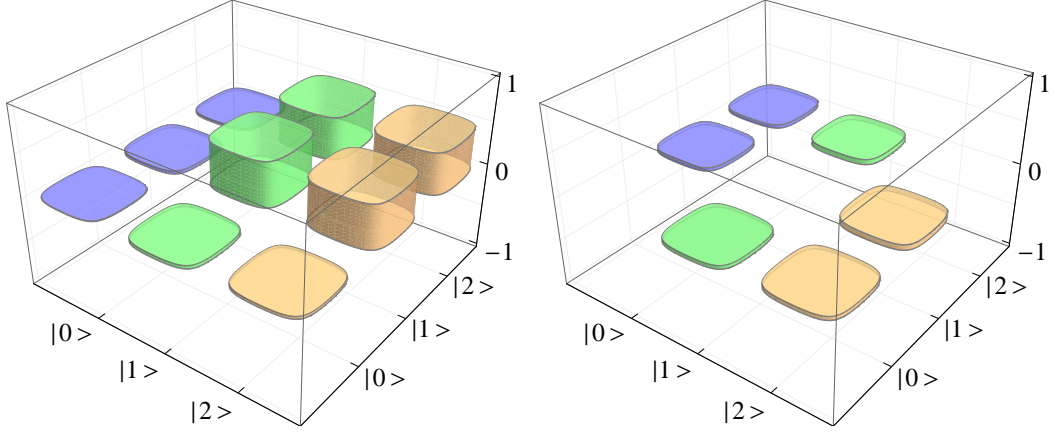
### Playing the quantum game

For quantum game the qutrit must be set in the state  $|\psi_A\rangle$ , which is done by opening all the slits and setting the initial direction of the propagating photon to be normal i. e.  $\alpha = 0$  with respect to the slits. Now, if Bob decides to check whether the photon passes through the slit number 0, he puts his detector there. His detector clicks resulting in losing the photon from the system, otherwise

(a)  $|\psi_1\rangle = (|0\rangle + |1\rangle + |2\rangle)/\sqrt{3}$



(c)  $|\psi_2\rangle = (|1\rangle + |2\rangle)/\sqrt{2}$



(e)  $|\psi_3\rangle = (|1\rangle + \exp(i\beta)|2\rangle)/\sqrt{2}$

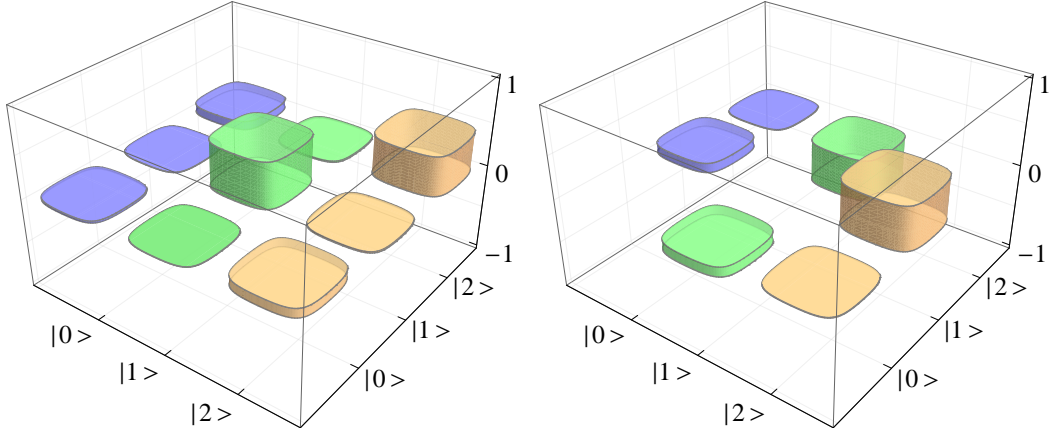


FIG. 2: Pure states reconstructed based on experimental data shown in table I. Left (right) column depict real (imaginary) part of reconstructed density matrix.

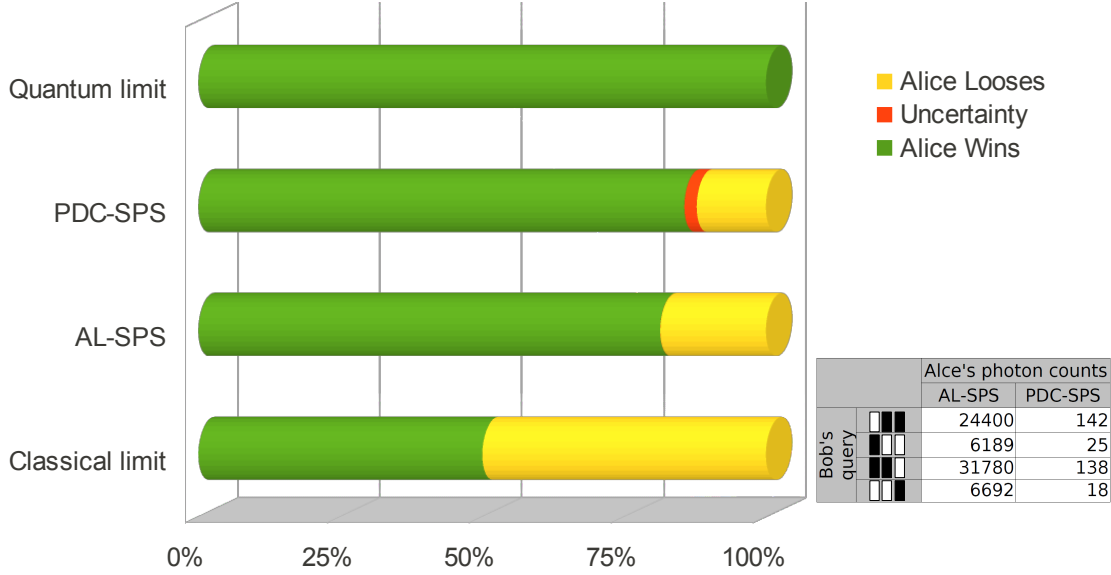


FIG. 3: Experimental and theoretical, best classical and best quantum winning trials in the quantum game. Table shows the number of photon counts measured by Alice for each of possible Bob action and his measurement outcome. For AL-SPS photons were collected for 2 s. In case of PDC-SPS the coincidences were measured for 1 min with the coincidence window set to 1 ns.

the photon goes through open slits. This can be simulated by blocking slit number 0, which will allow us to simulate all those cases when Bob's detector did not click. On the other hand, the case of finding a photon in slit number 0 can be simulated by closing all others. Alice has a way to say if Bob found the photon or not by placing the detector in far field plane in the position corresponding to POVM element  $M_{\text{ff}}(\theta = \pi)$  what is equivalent to projective measurement on the state  $|\psi_{Am}\rangle = (|0\rangle - |1\rangle + |2\rangle)/\sqrt{3}$ . If her detector does not register a photon she cancels the trial otherwise she is sure that Bob found the photon. Similar reasoning can be done for the case of Bob looking for the photon passing through the slit number 2.

We simulated all possible scenarios of Bob's measurement using PDC-SPS and AL-SPS. The measured photon counts are presented in table as an inset of Fig. 3. First row comprise single ptohon counts acquired in 2 seconds, whereas the second row presents the number of herladed photons measured in 1 min. In the perfect case one expects no counts if two slits are open. Here the nonzero photon count is attributed to: finite size of a multimode fiber core, dark counts and a background noise. Even this practical limitations of our experimental setup give Alice 87% chance to win using PDC-SPS and 82% using AL-SPS. This is much better that classical strategy and close to ideal quantum one, see Fig. 3 for comparison.



## DISCUSSION

We experimentally presented a simple way to implement a three level system into a single photon's spatial degree of freedom, which allowed us to perform pure state tomography and simulate the Aharon-Vaidman quantum game. The encoding part resorts to the Young type experiment where a photon passes through 3 slits which defines its state. Controlling an initial propagation direction of a photon and configuration of the slits it was possible to encode certain class of states. However, there is no fundamental limit in terms of possible states that can be encoded as one can introduce attenuators and phase shifters in front of each slit that can extend the family of the possible states. In addition, it is noteworthy that this setup is scalable as the number of slits can in principle be increased and the measurements can be performed. In general using  $m$  slits one can encode  $m$  level system and perform POVM of rank  $3m - 2$ , which is topographically complete for pure states.

## ACKNOWLEDGEMENTS

PK acknowledges fruitful discussions with Rafal Demkowicz-Dobrzanski from Warsaw University and support by the Foundation for Polish Science TEAM project cofinanced by the EU European Regional Development Fund. AC acknowledges support by MICINN Project FIS2008-05596 and the Wenner-Gren Foundation. The authors acknowledge insightful discussions with Christopher Fuchs, Matthew Graydon and Geo Noel Tabia from Perimeter Institute and funding from NSERC (CGS, QuantumWorks, Discovery, USRA), Ontario Ministry of Research and Innovation (ERA program), CIFAR, Industry Canada and the CFI.

---

\* Electronic address: piotr.kolenderski@uwaterloo.ca

- [1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).
- [2] H. J. Briegel, D. E. Browne, W. Dur, R. Raussendorf, and M. Van den Nest, Nat. Phys. **5**, 19 (2009).
- [3] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, Rev. Mod. Phys. **74**, 145 (2002).
- [4] N. Gisin and R. Thew, Nat. Photon. **1**, 165 (2007).
- [5] M. Kacprowicz, R. Demkowicz-Dobrzanski, W. Wasilewski, K. Banaszek, and I. Walmsley, Nat. Photon. **4**, 357 (2010).
- [6] N. Aharon and L. Vaidman, Phys. Rev. A **77**, 052310 (2008).
- [7] L. DiCarlo, J. M. Chow, J. M. Gambetta, L. S. Bishop, B. R. Johnson, D. I. Schuster, J. Majer, A. Blais, L. Frunzio, S. M. Girvin, et al., Nature **460**, 240 (2009).
- [8] M. Rådmark, M. Żukowski, and M. Bourennane, New J. Phys. **11**, 103016 (2009).
- [9] C. Negrevergne, T. S. Mahesh, C. A. Ryan, M. Ditty, F. Cyr-Racine, W. Power, N. Boulant, T. Havel, D. G. Cory, and R. Laflamme, Phys. Rev. Lett. **96**, 170501 (2006).
- [10] L. Neves, G. Lima, J. G. Aguirre Gómez, C. H. Monken, C. Saavedra, and S. Pádua, Phys. Rev. Lett. **94**, 100501 (2005).
- [11] G. Lima, L. Neves, E. S. G. R. Guzmán, W. A. T. Nogueira, A. Delgado, A. Vargas, and C. Saavedra, Opt. Express, **19**, 3542 (2010).

- [12] R. T. Thew, A. Acín, H. Zbinden, and N. Gisin, Quant. Inf. Proc. Vol. **4**, 93 (2004).
- [13] R. T. Thew, A. Acín, H. Zbinden, and N. Gisin, Phys. Rev. Lett. **93**, 010503 (2004).
- [14] A. Maser, R. Wiegner, U. Schilling, C. Thiel, and J. von Zanthier, Phys. Rev. A **81**, 053842 (2010).
- [15] H. Hossein-Nejad, R. Stock, and D. F. V. James, Phys. Rev. A **80**, 022308 (2009).
- [16] U. Sinha, C. Couteau, T. Jennewein, R. Laflamme, and G. Weihs, Science **329**, 418 (2010).
- [17] A. Orłowski and H. Paul, Phys. Rev. A **50**, 921 (1994).
- [18] K. Banaszek, G. M. D’ariano, M. G. Paris, and M. F. Sacchi, Phys. Rev. A **61**, 010304 (2000).